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✓PIECE-RATE INCENTIVE SCHEMES

by

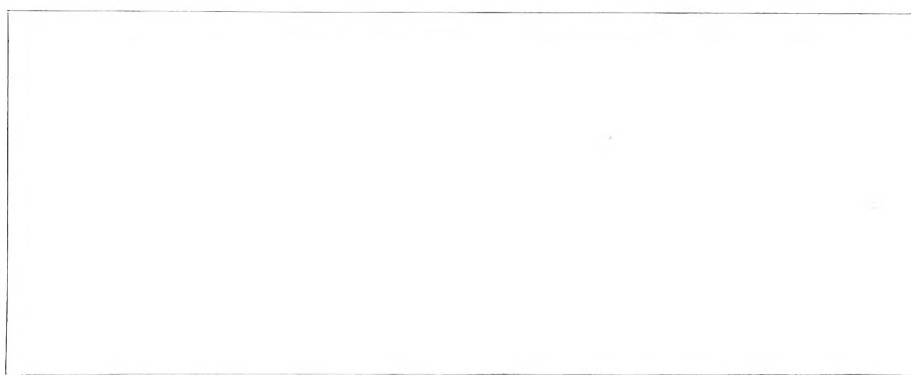
Robert Gibbons*

Number 424

July 1985
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**massachusetts
institute of
technology**

**50 memorial drive
cambridge, mass. 02139**



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Robert Gibbons*

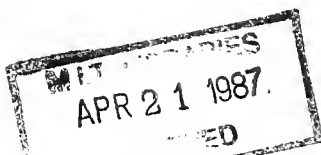
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Abstract

This paper uses recent results from incentive theory to study heretofore informal critiques of piece-rate compensation schemes. The informal critiques are based on the history of failed attempts to install piece-rate compensation schemes at the turn of the century. The formal analysis emphasizes the importance of information and commitment in contracting. In particular, in a work environment characterized by adverse selection and moral hazard, if neither the firm nor the worker can commit to future behavior, then no compensation scheme, piece-rate or otherwise, can induce the worker not to restrict output. This striking result is based on the path-breaking work of Laffont and Tirole (1985b).



1. Introduction

The incentive properties of piece-rate compensation schemes seem very attractive: workers are paid for the work they do, not the work they could have done, and this solves problems associated with both adverse selection and moral hazard. The inefficient signaling in Spence's (1973) model, for instance, would not occur if a piece rate equal to the market price of output were paid. Similarly, in an agency relationship with symmetric information, a piece rate equal to the market price of output induces a risk-neutral agent to provide the first-best level of effort.

But piece rates are far less prevalent in practice than this cursory analysis implies: Ehrenberg and Smith (1985, p. 344), for example, find that eighty-six percent of U.S. workers are paid either by the hour or by the month. This suggests that piece rates (whether linear or otherwise) often are either not feasible or not optimal. This paper assumes not only that piece rates are feasible but also that the necessary measurement of output is costless, thereby ruling out the plausible explanation of costly measurement in order to study alternative explanations based on information and commitment. The theme of the paper is that these explanations are well supported by the history of failed attempts to install piece-rate compensation schemes at the turn of the century.

Modern accounts of F.W. Taylor's scientific management often explain the difficulties the movement encountered in terms of asymmetric information. Edwards (1979), for instance, argues that piece-rate compensation schemes will be ineffective because management does not know how fast a job can be done and therefore cannot set the correct piece rate. And Clawson (1980)

then the firm can revise the piece rate based on first-period performance, as Clawson suggests, but the worker need not restrict output; and (3) if the firm cannot commit to the second-period contract and the worker cannot commit to stay with the firm for the second period, then no compensation scheme of any form can induce the worker not to restrict output. The last of these results is based on the path-breaking work of Laffont and Tirole (1985b).

2. Two Critiques of Piece Rates

This section argues that piece rates have two serious shortcomings. The first arises because workers have private information about the difficulty of their jobs. Edwards summarizes the historical record as follows:

[M]anagers' ability to control soldiering resulted from their inadequate knowledge of the actual techniques of production. Most of the specific expertise---for example, knowledge of how quickly production tasks could be done---resided in workers...

Piece-rates always carried the allure of payment for actual labor done (rather than labor power), thus promising an automatic solution to the problem of translating labor power into labor...[But] as long as management depended on its workers for information about how fast the job could be done... there was no way to make the piece-rate method deliver its promise. (pp. 98-9)

In the language of information economics, management faces both adverse-selection and moral-hazard problems: only workers know the difficulty of their jobs, and they can shirk so as to obscure this information from management. For risk-averse workers, of course, agency theory proves that piece rates typically are an inferior solution to the problem of moral hazard and risk sharing, and so presumably are an inferior solution to this more complicated problem as well.¹ Many jobs, however, simply do not involve a great deal of risk, which suggests that risk-aversion is not entirely responsible for the unpopularity of piece rates. In order to focus on

different culprits, this paper ignores the risk piece rates impose on workers by assuming that workers are neutral to income risk.

The second shortcoming of piece rates stems from the firm's opportunity to revise the rate over time. After discussing many case studies at length, Clawson concludes:

In theory, piecework was simple. The company set a fair price for each unit of completed work... and workers were paid according to their output. If workers could increase output, either by extra exertion or by improved methods of their own devising, they would receive higher wages... In practice, piecework never worked this way, since employers always cut the price they paid workers... Almost all employers insisted that they would never cut a price once it was set, yet every employer did cut prices... Unless workers collectively restricted output they were likely to find themselves working much harder, producing much more, and earning only slightly higher wages. (pp. 169-70)

If complete contracts could be written, the firm could commit to a fixed piece rate, but in practice the relevant contract is much too complex to write (not to mention to enforce) because the obvious simple contract will not suffice. As Clawson observes:

Employers could cut rates in dozens of ways other than changing the piece price for a worker who continued to perform the same operations. New workers could be assigned to the job at a lower rate while the old workers were transferred elsewhere, information about output on one job could be used to lower the initial price on new work, and any sort of minor change could be made the excuse for large price cuts. (p.170)

This paper captures these contractual difficulties in a dynamic model by allowing the firm no interperiod commitment opportunities and requiring it to be sequentially rational: in each period, the firm's action must be optimal from that point onward, as in a dynamic program.

3. The Static Model

This section uses a static model to formalize Edwards' critique: "As long as management depended on its workers for information about how fast the job

could be done...there was no way to make the piece-rate method deliver its promise."

To keep things simple, consider one firm employing one worker.² Output, y , is determined by the difficulty of the job, θ , and the effort the worker expends, a , according to

$$y = \theta + a,$$

where effort is chosen from $[0, \infty)$. Note that jobs with lower θ 's are more difficult.

Before contracting and production occur, the worker knows the difficulty of the job but the firm knows only that θ has distribution $F(\theta)$ on $[\underline{\theta}, \bar{\theta}]$. To simplify the exposition, the inverse of the hazard rate,

$$\frac{1-F(\theta)}{f(\theta)}$$

is assumed to decrease strictly in θ . Assumptions of this form are standard in the literature.³

The worker chooses effort to maximize the expectation of the separable utility function $u(w,a) = w - g(a)$, subject to the wage schedule $w(y)$ chosen by the firm. The disutility of effort, g , is increasing, strictly convex, and (without loss of generality) satisfies $g(0) = 0$. Also, the analysis is simplified by the stronger but not counter-intuitive assumptions that $g'(0)=g''(0)=0$ and $g''' > 0$, which guarantee that the optimal compensation scheme induces positive effort no matter what the job's difficulty, and that $g'(a)$ approaches infinity as a approaches infinity, which guarantees that the relevant first-order conditions have solutions. In particular, the efficient (or first-best) effort level solves $g'(a)=1$ and will be denoted by a_{fb} in what follows. Finally, the worker's next-best alternative is assumed to be unemployment, which is characterized by zero wage and zero effort, and

therefore zero utility.⁴

The firm's only cost is its wage bill, so it chooses a wage schedule to maximize expected profit, $E[y - w(y)]$, subject to optimizing behavior by the worker.⁵ In this one-period problem, the revelation principle (Myerson (1979), Dasgupta, Hammond, and Maskin (1979)) states that the firm's choice of a wage schedule $w(y)$ is equivalent to the choice of a suitable pair of functions $y(\theta)$ and $w(\theta)$ in a direct-revelation game: the firm chooses $\{y(\theta), w(\theta)\}$ to maximize expected profit

$$(E\Pi) \quad \int_{\underline{\theta}}^{\bar{\theta}} [y(\theta) - w(\theta)] f(\theta) d\theta$$

subject to incentive compatibility, individual rationality, and the feasibility constraint that $y(\theta) \geq \theta$ (since $a > 0$).⁶ To express the incentive-compatibility and individual-rationality constraints, define $U(\tilde{\theta}, \theta)$ to be the utility of a worker of type θ who reports type $\tilde{\theta}$ in the direct revelation game:

$$U(\tilde{\theta}, \theta) \equiv w(\tilde{\theta}) - g[y(\tilde{\theta}) - \theta].$$

Also, let $U(\theta)$ denote $U(\theta, \theta)$, the utility from truthful reporting. Then the incentive-compatibility constraint is

$$(IC) \quad U(\theta) \geq U(\tilde{\theta}, \theta) \text{ for all } \theta, \tilde{\theta},$$

and the individual-rationality constraint is

$$(IR) \quad U(\theta) \geq 0 \text{ for all } \theta.$$

In these terms, the firm's problem is to choose $\{y(\theta), w(\theta)\}$ to maximize (E\Pi) subject to (IC), (IR), and the feasibility constraint $y(\theta) \geq \theta$.

Lemmas 1 and 2 and Proposition 1 solve this problem. The techniques in the lemmas are due to Mirrlees (1971) and Myerson (1981). Results similar to the Proposition have been derived by many; this particular result is given in Sappington (1983) and Laffont and Tirole (1985a). Since the proofs of these results are not new, they are relegated to the Appendix. Corollary 1 then concludes that the solution is not a linear piece-rate compensation scheme. Finally, three Remarks following Corollary 1 interpret the results.

LEMMA 1. The output and wage functions $\{y(\theta), w(\theta)\}$ satisfy (IC) and (IR) if and only if

$$(a) \quad U(\theta) = U(\underline{\theta}) + \int_{\underline{\theta}}^{\theta} g'[y(\theta) - \theta] d\theta,$$

$$(b) \quad U(\underline{\theta}) > 0, \text{ and}$$

$$(c) \quad y(\theta) \text{ is nondecreasing.}$$

The most important part of the lemma is condition (a). The intuition behind this result is akin to that behind a separating equilibrium in Spence's signaling model. Here a worker in a job of difficulty θ must be persuaded not to claim that the job is more difficult, $\tilde{\theta} < \theta$; there the incentive is to overstate one's ability. By the envelope theorem, for very small lies (i.e., as $\tilde{\theta}$ approaches θ from below), the worker stands to gain $g'[y(\theta) - \theta]$ from lying; in the signaling model, the analogous gain is the marginal product of ability. Condition (a) dictates that the worker's equilibrium utility must include a bribe of this size to prevent such a lie; in the signaling model, the gain is matched by the cost of the extra education necessary to persuade the market that one's ability is as claimed. Condition (b) is clearly necessary for individual rationality, and (c) is a

second-order condition in the incentive-compatibility problem.

Given Lemma 1, substituting (a) and the definition of $U(\theta)$ into (E1) and changing the order of integration yields a convenient restatement of the firm's objective.

LEMMA 2. The firm's problem can be reduced to choosing $y(\theta)$ and $U(\underline{\theta})$ to maximize

$$(1) \quad -U(\underline{\theta}) + \int_{\underline{\theta}}^{\bar{\theta}} \left\{ y(\theta) - g[y(\theta) - \theta] - \left[\frac{1-F(\theta)}{f(\theta)} \right] g'[y(\theta) - \theta] \right\} f(\theta) d\theta,$$

subject to (b), (c), and $y(\theta) \geq \theta$.

PROPOSITION 1. At the optimum the firm sets $U(\underline{\theta})=0$ and chooses $y^*(\theta)$ to solve

$$(2) \quad 1 - g'[y - \theta] - \left[\frac{1-F(\theta)}{f(\theta)} \right] g''[y - \theta] = 0.$$

The resulting effort level, $a^*(\theta) \equiv y^*(\theta) - \theta$, is strictly positive, strictly increasing, and equals a_{fb} only at $\bar{\theta}$.

The intuition behind Proposition 1 is straight forward: In the standard agency problem, if the agent is risk-neutral then the principal sells the firm for price p by offering the contract $\tilde{w}(y) = y - p$, and this induces the efficient effort level, a_{fb} . Here the problem is that only the agent knows how much the firm (or, more intuitively, the job) is worth. For a fixed price p there exists a type $\theta(p)$ such that all types $\theta < \theta(p)$ do not take the contract, while all types $\theta > \theta(p)$ take the contract, put forth the efficient effort level, and earn rents. Keeping the cutoff-type $\theta(p)$ constant, the

envelope theorem dictates that the second-order loss incurred in moving away from efficient effort is more than covered by the accompanying first-order reduction in the rents earned by those who take the contract. At the same time, it is efficient to reduce $\theta(p)$.

Mathematically, the optimal contract given by (2) is simply the first-order condition for the pointwise maximization of (1). It trades off productive efficiency against lost rents, and has the familiar property that only the top type, $\bar{\theta}$, puts forth the efficient level of effort.

COROLLARY 1. A linear piece rate is not the optimal compensation scheme. Indeed, the optimum is nowhere linear.

PROOF: Recovering $w^*(\theta)$ from the definition of $U(\theta)$ and (a) yields

$$(3) \quad w^*(\theta) = U(\underline{\theta}) + g[y^*(\theta) - \underline{\theta}] + \int_{\underline{\theta}}^{\theta} g'[y^*(\theta') - \theta'] d\theta'.$$

In a linear piece rate, $dw/dy = (dw/d\theta)(d\theta/dy)$ must be constant. But $dw/d\theta = g'(y^* - 1) + g' = g'y'$, so $dw/dy = g'$, and Proposition 1 shows that $y^*(\theta) - \theta$ is strictly increasing, so $dw^*/dy = g'[y^*(\theta) - \theta]$ is nowhere constant. Q.E.D.

Remark 1. It is possible to interpret $\{y^*(\theta), w^*(\theta)\}$ as the upper envelope of a menu of linear compensation schemes among which workers select. (As with Lemma 1, the intuition for this parallels that for a separating equilibrium in a signaling model, or in any other self-selection model based on the familiar condition on the cross-partial derivative of the relevant utility function.) Notice that the best response of a worker of type θ to the linear compensation scheme $\tilde{w}(y) = by + c$ is the effort $a(b)$ that solves $g'(a) = b$. Since the effort induced by $\{y^*(\theta), w^*(\theta)\}$ is $y^*(\theta) - \theta$, the linear

compensation scheme designed for worker θ has slope $b(\theta) = g'[y^*(\theta) - \theta]$ and intercept $c(\theta) = w^*(\theta) - b(\theta)y^*(\theta)$. Such a menu of linear compensation schemes induces the worker to reveal the job's difficulty; this will not be possible in the dynamic model analyzed in next section.

Remark 2. Suppose the firm chooses a linear compensation scheme---that is, a single price per unit of output that applies to workers of all types. (This is analogous to choosing a two-part tariff when optimality requires a nonlinear price schedule.) The qualitative properties associated with the contract $\tilde{w}(y) = y - p$ reappear if the firm offers $\tilde{w}(y) = by + c$. As noted above, every worker who chooses to work will supply the effort $a(b)$ that solves $g'(a) = b$, while workers satisfying $\{b[\theta + a(b)] + c\} - g[a(b)] < 0$, or

$$\theta < \tilde{\theta} \equiv \frac{g[a(b)] - c}{b} - a(b),$$

will choose not to work. Assuming $\tilde{\theta} \in (\underline{\theta}, \bar{\theta})$, effort under a piece-rate compensation scheme is a step function: it is zero for $\theta \in [\underline{\theta}, \tilde{\theta})$ and $a(b)$ for $\theta \in [\tilde{\theta}, \bar{\theta}]$. One of the virtues of the optimal contract is that $a^*(\theta) \equiv y^*(\theta) - \theta$ is strictly positive for all θ .

Remark 3. Edwards describes a system of "differential rate piece-work" designed by Taylor to strengthen workers' incentives for effort (p.100). According to this system, a contract should be piecewise linear with kinks, or even jumps, so that productive workers benefit from a higher rate once their output surpasses a certain standards. In this connection, it is interesting that the optimal contract is nowhere linear.

4. The Dynamic Model

This section uses a dynamic model to formalize Clawson's critique: "Unless workers collectively restricted output they were likely to find themselves working much harder, producing much more, and earning only slightly higher wages." This is the "ratchet effect" analyzed by Freixas, Guesnerie, and Tirole (1985) for the case of linear incentive schemes when θ can take only two values. The goal of the section is to study the role of commitment in a dynamic model. The central result is that if the firm cannot commit to the second-period contract and the worker cannot commit to stay with the firm for the second period, then no compensation scheme of any form can induce the worker not to restrict output. Note that this result does not say that piece rates are suboptimal. Rather, it formally identifies an unavoidable inefficiency, and thereby provides support for earlier informal accounts.

Let there be two periods of work, each of which is identical to that described in the previous section. In period t , the worker's output, y_t , is determined by the difficulty of the worker's job, θ , and the effort the worker expends that period, a_t , according to

$$y_t = \theta + a_t.$$

Notice that θ is constant over time. As before, the worker knows the difficulty of the job but the firm does not. Before period one, the firm believes that θ has distribution $G(\theta)$ on $[\theta^-, \theta^+]$. Its posterior given first-period output may be more refined, however. Denoting the posterior by $F(\theta)$ on the sub-interval $[\underline{\theta}, \bar{\theta}]$ of $[\theta^-, \theta^+]$, the analysis of Section 3 applies to the second-period subgame here, unless the firm can commit in advance to do otherwise.

The worker's von Neumann-Morgenstern preferences are represented by the discounted sum of each period's utility, which is itself a separable function of consumption, c_t , and effort:

$$\sum_{t=1}^2 \beta^{t-1} [c_t - g(a_t)].$$

The subjective discount factor, β , is assumed to be determined by the market interest rate, r , according to $\beta = 1/(1+r)$. Since the worker is neutral to income risk and discounts at the market rate, borrowing and saving can be ignored. The firm shares the discount factor β and maximizes the expected discounted sum of each period's profit,

$$E \sum_{t=1}^2 \beta^{t-1} [y_t - \tilde{w}_t(y_t)],$$

where $\tilde{w}_t(\cdot)$ is the wage schedule the firm offers the worker for period t .

The results in this dynamic model depend critically on the firm's and the worker's commitment opportunities. Following a long tradition in labor-market models, one could assume that the worker is free to quit the job, at no penalty, at any time; this is frequently described as a prohibition of involuntary servitude. As before, the worker's next-best alternative is assumed to be unemployment, which yields zero utility. And on the firm's side, the difficulties in writing, not to mention enforcing, the appropriate two-period contract may force the firm to offer only single-period contracts. (Clawson's persuasive description of these difficulties is quoted in Section 2.) Sequential rationality in the absence of commitment forces the firm to choose $\tilde{w}_2(\cdot)$ to maximize $E\{y_2 - \tilde{w}_2(y_2)\}$ with respect to its beliefs about θ conditioned on the observed value of y_1 .

It is intuitive (but see Laffont and Tirole (1985b) for a proof) that if the firm can commit to any second-period behavior it chooses, then the two-period optimum simply repeats the static optimal compensation scheme. The absence of commitment opportunities has a profound effect, however. To see this, suppose there is a separating equilibrium in the first period; that is, suppose that each y_1 the firm could observe results in degenerate posterior beliefs about θ . Then (after the obvious changes in notation) the analysis of Section 3 can be applied to the second-period subgame by letting $\tilde{\theta} = \theta$. This yields $g' = 1$ in (2), so effort is at the first-best level, a_{fb} . Output is therefore $y_2(\theta) = a_{fb} + \theta$, and (3) then indicates that $\tilde{w}_2(\theta) = g(a_{fb})$, independent of θ . On the equilibrium path, therefore, the worker gets zero utility in the second period following a separating equilibrium in the first period because the firm confiscates all the surplus. (The same result follows from the agency contract $\tilde{w}(y) = y - p$ discussed following Proposition 1: when θ is known, the principal can calculate that p should be $\theta + a_{fb} - g(a_{fb})$, leaving the worker exactly zero surplus.) This is in the spirit of Clawson's contention that the firm will cut its piece rate, but applies more generally to any separating first-period compensation scheme.

Given this second-period behavior, optimal first-period behavior can be determined by modifying the static game to include the second-period payoffs. Following the definitions in Section 3, redefine $U(\tilde{\theta}, \theta)$ to be $U_1(\tilde{\theta}, \theta) + \beta U_2(\tilde{\theta}, \theta)$, where $U_1(\tilde{\theta}, \theta) = w_1(\tilde{\theta}) - g[y_1(\tilde{\theta}) - \theta]$ and

$$U_2(\tilde{\theta}, \theta) = \begin{cases} g(a_{fb}) - g(a_{fb} + \tilde{\theta} - \theta) & \text{if } \theta > \tilde{\theta}, \\ 0 & \text{if } \theta < \tilde{\theta}. \end{cases}$$

The form of $U_2(\tilde{\theta}, \theta)$ reflects the combination of the worker's opportunity to quit after the first period and the firm's attempt to extract all the surplus from the worker: based on the belief that the job difficulty is $\tilde{\theta}$, the firm will pay the minimum wage $g(a_{fb})$ and require the output $y_2(\tilde{\theta}) = \tilde{\theta} + a_{fb}$; the worker quits if the implied second-period utility is negative.

Continue to denote $U(\theta, \theta)$ by $U(\theta)$. Then the incentive-compatibility constraints are again given by (IC). Following the proof of Lemma 1, substitute the definition of $U(\tilde{\theta})$ into (IC):

$$U(\theta) - U(\tilde{\theta}) > g[y(\tilde{\theta}) - \tilde{\theta}] - g[y(\tilde{\theta}) - \theta] + \beta U_2(\tilde{\theta}, \theta).$$

Reversing the roles of θ and $\tilde{\theta}$ then yields

$$\begin{aligned} g[y(\theta) - \tilde{\theta}] - g[y(\theta) - \theta] - \beta U_2(\theta, \tilde{\theta}) &> U(\theta) - U(\tilde{\theta}) \\ &> g[y(\tilde{\theta}) - \tilde{\theta}] - g[y(\tilde{\theta}) - \theta] + \beta U_2(\tilde{\theta}, \theta). \end{aligned}$$

Take $\theta > \tilde{\theta}$, divide by $\theta - \tilde{\theta}$, and let $\theta \rightarrow \tilde{\theta}$. This yields

$$(4) \quad g'[y(\theta) - \theta] > U'(\theta) > g'[y(\theta) - \theta] + \beta g'(a_{fb}),$$

which is impossible. This proves Proposition 2, which is due to Laffont and Tirole (1985b).

PROPOSITION 2. If neither the firm nor the worker can commit in advance to second-period behavior, then there is no sequentially rational pair of contracts $\{\tilde{w}_1(y_1), \tilde{w}_2(y_1, y_2)\}$ that separates any interval of worker types in the first period.

The intuition behind Proposition 2 mimics that behind Lemma 1: a worker in a job of difficulty θ must be persuaded not to claim that the job is more

difficult, $\tilde{\theta} < \theta$, and this requires a bribe. In this two-period model, the bribe must be bigger than was necessary in Lemma 1, because the claim that the job has difficulty $\tilde{\theta}$ now stands to earn rents in both periods, especially because the firm cannot commit to its second-period behavior. The catch is that the necessary bribe is so large that, provided the worker can quit after the first period, it is now profitable for a worker in a job of difficulty $\tilde{\theta}$ to claim that the job is less difficult, $\theta > \tilde{\theta}$, pocket the bribe, and then quit. (In fact, this incentive is so strong that each type $\theta \in [\underline{\theta}, \bar{\theta}]$ will claim to be $\bar{\theta}$ in order to pocket the bribe.) This incentive-compatibility problem is described by the inequalities in (4): the first inequality concerns the incentive to claim that the job is less difficult, and then pocket the bribe and quit, which is why the $U_2(\theta, \tilde{\theta})$ term disappears, while the second concerns the incentive to claim that the job is more difficult, thereby earning the second-period rent $U_2(\tilde{\theta}, \theta)$.

Proposition 2 says that the firm cannot infer the job's difficulty from the observed first-period output. This means that if a worker in a job of difficulty θ produces y_1 , then there exists another job difficulty $\theta' \neq \theta$ such that the worker also would produce y_1 . (Strictly speaking, when the job difficulty is θ' the worker produces y_1 with positive probability, perhaps less than one.) Denoting the required effort levels by a_1 and a'_1 , respectively, yields $\theta + a_1 = \theta' + a'_1$. Without loss of generality, let $\theta > \theta'$. Then $a_1 < a'_1$. This proves the main result of the paper:

COROLLARY 2. Piece-rate compensation schemes will not "translate labor power into labor" because workers will restrict output, in the sense that workers

in less difficult jobs often will produce no more than workers in more difficult jobs.

Proposition 2 makes strong use of the assumption that the worker can quit after the first period. Other assumptions have been studied by Baron and Besanko (1985) and Lazear (1985). Baron and Besanko work in terms of a direct-revelation game and impose the constraint that the worker is forced to accept a second-period contract that would yield the reservation utility (here zero) if the true type were the type announced in the first period. Lazear works with indirect mechanisms and makes the related assumption that the worker is committed to staying with the firm in the second period. An example shows what an important difference this kind of assumption makes.

For simplicity, assume that $\beta=1$. Consider the pair of contracts

$$\tilde{w}_1(y_1)=2y_1-g(a_{fb}) \text{ and } \tilde{w}_2(y_1,y_2)=y_2-y_1+g(a_{fb}).$$

These contracts are sequentially rational for the firm and induce the first-best effort level in both periods, provided the worker is committed to staying with the firm in the second period, as assumed by Lazear.

(Similarly, if the firm assumes that the worker chooses $a_1=a_{fb}$ then the observed first-period output y_1 is equivalent to the announced type $\tilde{\theta}=y_1-a_{fb}$. Based on this calculation of an announced type, these contracts also induce the first-best effort level in both periods if the worker is committed as described by Baron and Besanko.) If the worker can quit, however, then the optimal effort then the optimal effort strategy is to choose a_1^* to solve $g'(a)=2$ and then quit, yielding utility

$$U_1^* = 2(\theta+a_1^*)-g(a_{fb})-g(a_1^*),$$

rather than the utility that follows from $a_1=a_2=a_{fb}$,

$$U_1 = 2(\theta + a_{fb}) - 2g(a_{fb}).$$

A little algebra shows that $U_1^* > U_1$ if and only if

$$2 > \frac{g(a_1^*) - g(a_{fb})}{a_1^* - a_{fb}},$$

which follows from the convexity of $g(\cdot)$ and the definitions of a_1^* and a_{fb} .

Returning to Proposition 2, one should not conclude that (when the stated assumptions hold) piece rates will not be observed: the result does not say that piece rates are not optimal, but rather that it is not feasible for piece rates to induce workers to reveal their private information through their performance. When the uncertainty about θ is large, these restrictions in output may be sufficiently costly that piece rates will be inferior to time rates; Lazear considers several other factors that influence this comparison.

APPENDIX

LEMMA 1. The output and wage functions $\{y(\theta), w(\theta)\}$ satisfy (IC) and (IR) if and only if

$$(a) \quad U(\theta) = U(\underline{\theta}) + \int_{\underline{\theta}}^{\theta} g'[y(\theta') - \theta'] d\theta',$$

$$(b) \quad U(\underline{\theta}) > 0, \text{ and}$$

$$(c) \quad y(\theta) \text{ is nondecreasing.}$$

PROOF: Only if. Substituting the definition of $U(\tilde{\theta})$ in (IC) yields

$$U(\theta) - U(\tilde{\theta}) > g[y(\tilde{\theta}) - \tilde{\theta}] - g[y(\tilde{\theta}) - \theta],$$

and reversing the roles of θ and $\tilde{\theta}$ yields

$$(A) \quad g[y(\theta) - \tilde{\theta}] - g[y(\theta) - \theta] > U(\theta) - U(\tilde{\theta}) > g[y(\tilde{\theta}) - \tilde{\theta}] - g[y(\tilde{\theta}) - \theta].$$

Take $\theta > \tilde{\theta}$, divide by $\theta - \tilde{\theta}$, and let $\theta \rightarrow \tilde{\theta}$ in (A). This yields

$$U'(\theta) = g'[y(\theta) - \theta],$$

which implies (a). Clearly, (IR) implies (b). And finally, take $\theta > \tilde{\theta}$ and suppose for contradiction that $y(\tilde{\theta}) > y(\theta)$. By the convexity of g , if $\Delta > 0$ then $g(\delta + \Delta) - g(\delta)$ increases in δ , so

$$g[y(\tilde{\theta}) - \tilde{\theta}] - g[y(\tilde{\theta}) - \theta] > g[y(\theta) - \tilde{\theta}] - g[y(\theta) - \theta],$$

which contradicts (A).

If. Since $g' > 0$, (a) and (b) imply (IR). For (IC), use (a) to substitute

$$U(\underline{\theta}) + \int_{\underline{\theta}}^{\tilde{\theta}} g'[y(\theta') - \theta'] d\theta'$$

for $U(\tilde{\theta})$ in

$$U(\tilde{\theta}, \theta) = U(\tilde{\theta}) + g[y(\tilde{\theta}) - \tilde{\theta}] - g[y(\tilde{\theta}) - \theta].$$

This yields

$$U(\tilde{\theta}, \theta) = U(\theta) + \int_{\theta'=\theta}^{\tilde{\theta}} \{g'[y(\theta') - \theta'] - g'[y(\tilde{\theta}) - \theta']\} d\theta',$$

which implies (IC) because (c) and the convexity of g guarantee that the integrand is negative for $\tilde{\theta} > \theta$ and positive for $\theta > \tilde{\theta}$ (in which case the limits of integration must be reversed).

Q.E.D.

LEMMA 2. The firm's problem can be reduced to choosing $y(\theta)$ and $U(\underline{\theta})$ to maximize

$$(1) \quad -U(\underline{\theta}) + \int_{\theta=\underline{\theta}}^{\bar{\theta}} \{y(\theta) - g[y(\theta) - \theta] - \left[\frac{1-F(\theta)}{f(\theta)}\right] g'[y(\theta) - \theta]\} f(\theta) d\theta,$$

subject to (b), (c) and $y(\theta) \geq \theta$.

PROOF: By the definition of $U(\theta)$, $w(\theta) = U(\theta) + g[y(\theta) - \theta]$, where $U(\theta)$ is given by (a) in Lemma 1. Therefore

$$\int_{\theta=\underline{\theta}}^{\bar{\theta}} w(\theta) f(\theta) d\theta = U(\underline{\theta}) + \int_{\theta=\underline{\theta}}^{\bar{\theta}} \{g[y(\theta) - \theta] + \left[\frac{1-F(\theta)}{f(\theta)}\right] g'[y(\theta) - \theta]\} f(\theta) d\theta$$

after reversing the order of integration in the double integral.

PROPOSITION 1. At the optimum, the firm sets $U(\underline{\theta}) = 0$ and chooses $y^*(\theta)$ to solve

$$(2) \quad 1 - g'[y - \theta] - \left[\frac{1-F(\theta)}{f(\theta)}\right] g''[y - \theta] = 0.$$

The resulting effort level, $a^*(\theta) \equiv y^*(\theta) - \theta$, is strictly positive, strictly increasing, and equals a_{fb} only at $\bar{\theta}$.

PROOF: It suffices to show that for each θ the solution to (2) maximizes the kernel in (1),

$$y - g[y - \theta] - \left[\frac{1 - F(\theta)}{f(\theta)} \right] g'[y - \theta],$$

subject to $y(\theta)$ being nondecreasing and $y(\theta) > \theta$. Since this kernel is concave (because $g'' > 0$), the solution to (2) yields the unconstrained maximum. As for the effort level, $a(\theta) = y(\theta) - \theta$ is strictly increasing (and hence $y(\theta)$ is nondecreasing, as required) because implicitly differentiating (2) yields

$$y' - 1 = \frac{\left[\frac{d}{d\theta} \left(\frac{1 - F(\theta)}{f(\theta)} \right) \right] g''[y - \theta]}{g''[y - \theta] + \left(\frac{1 - F(\theta)}{f(\theta)} \right) g'''[y - \theta]} > 0,$$

since $[1 - F(\theta)]/f(\theta)$ strictly decreases in θ and the denominator is positive because of the second-order condition. Also, $a(\theta)$ is strictly positive (and hence $y(\theta) > \theta$, as required) because the lefthand side of (2) is positive at $y = \theta$:

$$1 - g'(0) - \left(\frac{1 - F(\theta)}{f(\theta)} \right) g''(0) > 0$$

because $g'(0) = g''(0) = 0$. Finally, substituting $\theta = \bar{\theta}$ into (2) yields $1 - F(\bar{\theta}) = 0$ and $g' = 1$, so $a^*(\bar{\theta}) = a_{fb}$. Q.E.D.

¹ It is rare but possible for a linear contract with a positive intercept to be optimal. This follows from the proposition that any monotone sharing rule is optimal for some special case of the agency problem.

² Alternatively, there could be as many workers as there are jobs in the firm provided the jobs have independent difficulties, and there could be many firms, subject to the same proviso. What is important is that no two workers share the same private information, for if they did then competitive compensation schemes might help extract it from them, and these are beyond the scope of the paper.

³ See, for instance, Baron and Besanko (1984), who list many familiar distributions that satisfy a related condition. The analysis can proceed without this assumption but at some technical expense; see Myerson (1981) and Baron and Myerson (1982).

⁴ Next-best alternatives other than unemployment are possible. For instance, self-employment could generate the reservation utility \bar{U} , which could be normalized to zero. In this case, however, it would be important that the worker not have access to the firm's technology, since this would vitiate the problem of private information.

⁵ As it stands, this is a model of a competitive firm facing a price of one. The model fits a wide variety of product markets, however, since the notion of output can be suppressed and y can be interpreted as revenue. Indeed,

since the firm will make profits in what follows, an imperfectly competitive interpretation is more natural.

⁶ The Revelation Principle guarantees an equivalence between direct and indirect mechanisms. This works as follows: If the firm chooses a compensation scheme $\tilde{w}(y)$, a worker in a job of difficulty θ will choose effort to maximize $\tilde{w}(y) - g(a)$ subject to $y = \theta + a$. Let the optimal effort choice be $\tilde{a}(\theta)$. Then output will be $y(\theta) = \theta + \tilde{a}(\theta)$ and wages will be $w(\theta) = \tilde{w}(y(\theta))$. Thus, any compensation scheme $\tilde{w}(y)$ can be represented by the appropriate pair $\{y(\theta), w(\theta)\}$.

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